Quantitative Electrolysis: The Math (Or How I Learned to Stop Worrying and Love Dimensional Analysis)

In an electrolytic cell, the amount of product made is related to the number of electrons transferred. Essentially, the electrons are a reactant. To solve, use the formula for current from the periodic table and follow the following steps:

1. Convert your time, t, into seconds.

S (Seconds)

2. Multiply your time (sec) by the current (Amp) to solve for the charge (q) in Coulombs.

$$I = \frac{q}{t} \rightarrow q = t \cdot I$$

I = current (amperes, A)
q = charge (coulombs, C)
t = time (sec)

$$\text{Amp} = \frac{C}{sec}$$

$\times A \ (\text{amp} = \frac{C}{sec})$

3. Divide by Faraday's constant (given on the formula chart) to convert Coulombs into moles of electrons.

$$q / F = \text{mol } e^-$$

Faraday's constant, F, is the charge on 1 mole of electrons.

F = 96, 485 C/mol e^-

Use F = 100,000 (1E5) for multiple choice!

$\times \frac{1}{F} = \frac{\text{mol } e^-}{C}$

*<u>Note</u>: a "faraday" can be used to refer to the number of moles of electrons being transferred:

- "3 faradays" → 3 moles of e^- of transferred
- "0.25 faradays" → 0.25 moles of e^- of transferred

$\times \frac{1 \text{ mol metal}}{\# \text{ mol } e^-}$

4. Use the appropriate half-reaction to determine the moles of the metal formed.

For example,

$Au^{3+}(aq) + 3e^- \rightarrow Au(s)$

This indicates for every 3 moles of electrons gained, 1 mole of solid gold is formed.

5. Use molar mass to convert from moles to grams of metal.

$\times \frac{g}{mol}$

Example: Gold can be plated out of a solution containing Au^{3+} according to the half-reaction:

$$Au^{3+}(aq) + 3\ e^- \rightarrow Au(s)$$

What mass of gold (in grams) is plated by a 25 minute flow of 5.5 A current?

Solution: $$\boxed{25\ \cancel{min} \times \frac{60\ \cancel{s}}{1\ \cancel{min}}} \times \boxed{\frac{5.5\ \cancel{C}}{1\ \cancel{s}}} \times \boxed{\frac{1\ \cancel{mol\ e^-}}{96{,}485\ \cancel{C}}} \times \boxed{\frac{1\ \cancel{mol\ Au}}{3\ \cancel{mol\ e^-}}} \times \boxed{\frac{196.97\ g\ Au}{1\ \cancel{mol\ Au}}} = 5.6\ g\ Au$$

$$\underset{(sec)}{time} \times \underset{(Amp)}{current} \times \frac{1}{\mathcal{F}} \times \frac{solid\ metal}{mol\ e^-} \times molar\ mass\ (g/mol)$$

You will need to be able to do two basic calculations for quantitative electrolysis:

1. Given time (sec) and current (A), calculate mass (g).
2. Given mass (g) and current (A), calculate time required (sec).

Of course, there are endless variations of these two calculation types we can give you! 😉

Quick Trick to remember the order of steps to calculate mass of a metal produced (given time and current):

Are you SAFe? MMM.

Sec amp F mol molar mass

Going backwards? MMM, FASe ^^

Let's Try!

1. If 3.30 faraday of charge is passed through a solution of $Al_2(SO_4)_3$, what mass of aluminum is deposited?

$$\boxed{3.30\ \mathcal{F} \Rightarrow 3.30\ mol\ e^-} \times \boxed{\frac{1\ mol\ Al}{3\ mol\ e^-}} \times \boxed{\frac{26.98\ g\ Al}{1\ mol\ Al}} = \boxed{29.7\ g\ Al}$$

2. How long must a current of 5.00 A be applied to a solution of AgCN to produce 10.5 g silver metal?

$$\boxed{10.5\ g\ Ag \times \frac{1\ mol\ Ag}{107.87\ g\ Ag}} \times \boxed{\frac{1\ mol\ e^-}{1\ mol\ Ag}} \times \boxed{\frac{96{,}485\ C}{1\ mol\ e^-}} \times \boxed{\frac{1\ sec}{5.00\ C}} = \boxed{1{,}880\ sec}$$

(or 31.3 min)

3. Copper may be used for electroplating, starting with a solution of $Cu(NO_3)_2(aq)$.
   a. If a current of 10.0 amp is applied to the $Cu(NO_3)_2(aq)$ solution for 60.0 minutes, what mass of copper will be plated out? (Assume excess $Cu(NO_3)_2(aq)$).

$$\boxed{60.0\ min \times \frac{60\ sec}{1\ min}} \times \boxed{\frac{10.0\ C}{1\ sec}} \times \boxed{\frac{1\ mol\ e^-}{96{,}485\ C}} \times \boxed{\frac{1\ mol\ Cu}{2\ mol\ e^-}} \times \boxed{\frac{63.55\ g\ Cu}{1\ mol\ Cu}} = \boxed{11.9\ g\ Cu}$$

   b. How many moles of electrons must be transferred in this reaction to produce 5.16 g of copper metal?

$$\boxed{5.16\ g\ Cu \times \frac{1\ mol\ Cu}{63.55\ g\ Cu}} \times \boxed{\frac{2\ mol\ e^-}{1\ mol\ Cu}} = \boxed{0.162\ mol\ e^-}$$